Esercizio

Dato il campo di spostamento

$$\underset{\sim}{u}(\underset{\sim}{x}) = (0{,}1\,x_2 - 0{,}1\,x_1)\,\underset{\sim}{e_1} + (-0{,}1\,x_1 - 0{,}1\,x_2)\,\underset{\sim}{e_2}$$

1) calcolare il vettore traslazione e il vettore della rotazione rigida infinitesima
2) scrivere il tensore della deformazione
3) calcolare la distorsione angolare tra le fibre $\underset{\sim}{n} = (-\sqrt{2}/2, \sqrt{2}/2)$ e $\underset{\sim}{m} = (\sqrt{2}/2, \sqrt{2}/2)$
4) disegnare la configurazione deformata di un quadrato di lato unitario.

Svolgimento

$$\underset{\sim}{u}(\underset{\sim}{x}) = \underbrace{(0,1\,x_2 - 0,1\,x_1)}_{u_1}\,\underset{\sim}{e_1} + \underbrace{(-0,1\,x_1 - 0,1\,x_2)}_{u_2}\,\underset{\sim}{e_2}$$

1). Vettore Traslazione

$$u(x + dx) = u(x) + \underline{\underline{H}}\,\underline{dx}$$

$$u_0 = (0 \quad 0)^T$$

Esempio con vettore non nullo

$$\boxed{\begin{gathered} \underset{\sim}{u}(x) = (1 + 0,1\,x_2)\,\underset{\sim}{e_1} + (0,5 - 0,1\,x_1)\,\underset{\sim}{e_2} \\ u_0 = (1 \quad 0,5)^T \end{gathered}}$$

- Vettore rotazione rigida infinitesima

$$\underline{\underline{W}} = \frac{1}{2}(\underline{\underline{H}} - \underline{\underline{H}}^T)$$

$$\underline{\underline{H}} = \begin{pmatrix} u_{11} & u_{12} & u_{13} \\ u_{21} & u_{22} & u_{23} \\ u_{31} & u_{32} & u_{33} \end{pmatrix}$$

$$\underline{\underline{W}} = \begin{array}{c} \begin{array}{ccc} e_1 & e_2 & e_3 \end{array} \\ \begin{pmatrix} 0 & -\theta_3 & \theta_1 \\ \theta_3 & 0 & -\theta_2 \\ -\theta_1 & \theta_2 & 0 \end{pmatrix} \end{array} \qquad \underset{\sim}{\theta} = \begin{pmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \end{pmatrix}$$

$$\underset{=}{H} = \begin{pmatrix} -0,1 & 0,1 & 0 \\ -0,1 & -0,1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \Rightarrow \underset{=}{H} = \begin{pmatrix} -0,1 & 0,1 \\ -0,1 & -0,1 \end{pmatrix}$$

$$\underset{=}{W} = \frac{1}{2}\left[\begin{pmatrix} -0,1 & 0,1 \\ -0,1 & -0,1 \end{pmatrix} - \begin{pmatrix} -0,1 & -0,1 \\ 0,1 & -0,1 \end{pmatrix}\right] = \frac{1}{2}\begin{pmatrix} 0 & 0,2 \\ -0,2 & 0 \end{pmatrix} =$$

$$= \begin{array}{cc} \underset{\sim}{e_1} & \underset{\sim}{e_2} \\ \left(\begin{matrix} 0 \\ -0,1 \end{matrix}\right. & \left.\begin{matrix} 0,1 \\ 0 \end{matrix}\right) \end{array} \quad \text{AnTi sym}$$

$$\theta_1 = 0 \,, \quad \theta_2 = 0 \,, \quad \theta_3 = -0,1 \quad \Rightarrow \quad \underset{\sim}{\theta} = \begin{pmatrix} 0 \\ 0 \\ -0,1 \end{pmatrix}$$

2) Tensore della deformazione

$$\underline{\underline{E}} = \frac{1}{2}\left(\underline{\underline{H}} + \underline{\underline{H}}^T\right)$$

$$\underline{\underline{E}} = \frac{1}{2}\left[\begin{pmatrix} -0{,}1 & 0{,}1 \\ -0{,}1 & -0{,}1 \end{pmatrix} + \begin{pmatrix} -0{,}1 & -0{,}1 \\ 0{,}1 & -0{,}1 \end{pmatrix}\right] = \frac{1}{2}\begin{pmatrix} -0{,}2 & 0 \\ 0 & -0{,}2 \end{pmatrix} =$$

$$= \begin{pmatrix} -0{,}1 & 0 \\ 0 & -0{,}1 \end{pmatrix} \quad \text{Sym}$$

3) Scorrimento angolare γ_{12} fibre di direzioni

$\underset{\sim}{n} = (-\sqrt{2}/2 \quad \sqrt{2}/2)^T$ $\qquad$ $\underset{\sim}{m} = (\sqrt{2}/2 \quad \sqrt{2}/2)^T$

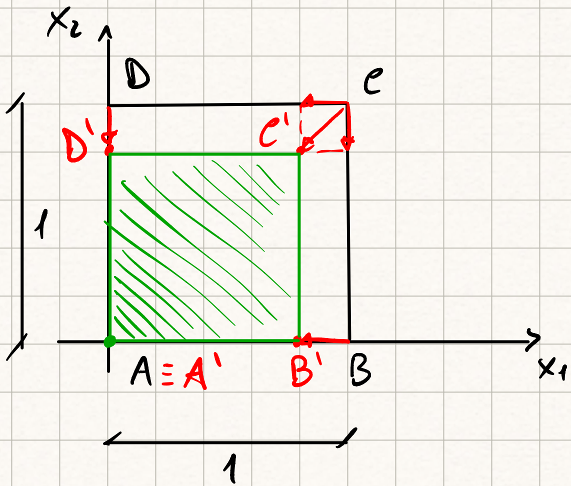
$$\gamma_{nm} = 2\,\underset{\sim}{m}^T \underset{\approx}{E}\, \underset{\sim}{n}$$

$$\gamma_{nm} = 2 \begin{pmatrix} \sqrt{2}/2 & \sqrt{2}/2 \end{pmatrix} \begin{pmatrix} -0{,}1 & 0 \\ 0 & -0{,}1 \end{pmatrix} \begin{pmatrix} -\sqrt{2}/2 \\ \sqrt{2}/2 \end{pmatrix} =$$

$$= 2 \begin{pmatrix} -0{,}1 \cdot \frac{\sqrt{2}}{2} & -0{,}1 \cdot \frac{\sqrt{2}}{2} \end{pmatrix} \begin{pmatrix} -\frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} \end{pmatrix} =$$

$$= 2 \left(0{,}1 \cdot \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{2}}{2} - 0{,}1\, \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{2}}{2} \right) = 0$$

a) Disegniamo la configurazione deformata



$$\underset{\sim}{u}(\underline{x}+d\underline{x}) = \underset{\sim}{u}(\underset{\sim}{x}) + \underline{\underline{H}}\, d\underset{\sim}{x}$$

$$= \underset{\sim}{u}(\underset{\sim}{x}) + \underline{\underline{W}}\, d\underset{\sim}{x} + \underline{\underline{E}}\, d\underset{\sim}{x}$$

$$\underset{\sim}{u}(\underline{x}+d\underline{x}) = \underline{\underline{E}}\, d\underset{\sim}{x}$$

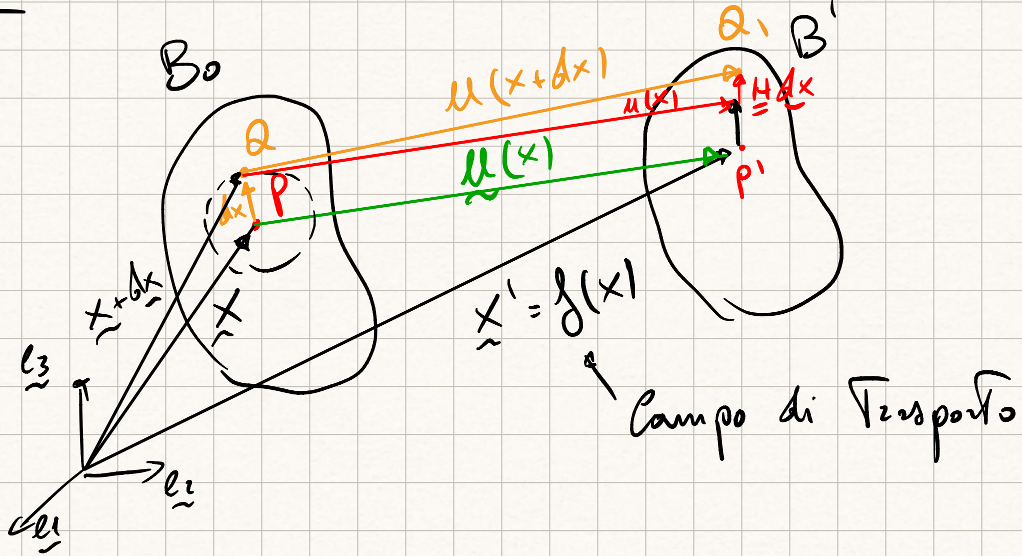
$$u(A) = \begin{pmatrix} -0,1 & 0 \\ 0 & -0,1 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$u(B) = \begin{pmatrix} -0,1 & 0 \\ 0 & -0,1 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} -0,1 \\ 0 \end{pmatrix}$$

$$u(C) = \begin{pmatrix} -0,1 & 0 \\ 0 & -0,1 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} -0,1 \\ -0,1 \end{pmatrix}$$

$$u(D) = \begin{pmatrix} -0,1 & 0 \\ 0 & -0,1 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ -0,1 \end{pmatrix}$$

Oss



$$u(x+dx) = u(x) + \underline{\underline{H}}\,\underset{\sim}{dx} + \cancel{o(\underset{\sim}{dx})}$$