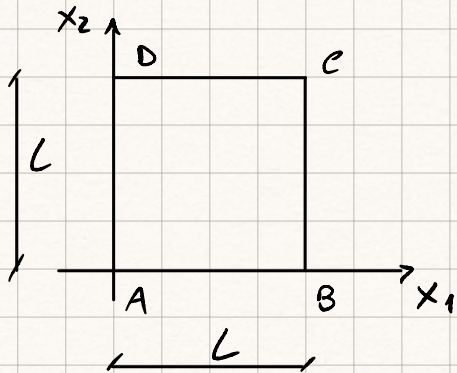


## Esercizio



$$\underline{\underline{f}}(x) = (x_1 - 0,2x_2)\underline{\underline{e}}_1 + (0,3x_1 + x_2)\underline{\underline{e}}_2 + x_3\underline{\underline{e}}_3$$

$$\lambda = 34615 \text{ MPa} \quad \mu = 23077 \text{ MPa}$$

$$L = 1000 \text{ mm}$$

Dato il campo di trasporto  $\underline{\underline{f}}$ , determinare:

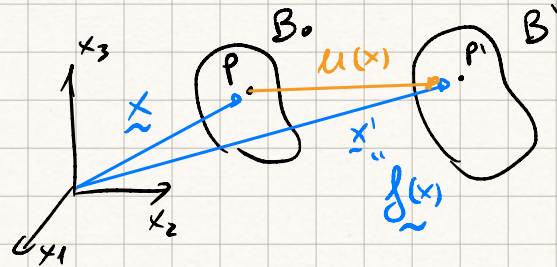
- 1) il campo di spostamento
- 2) il gradiente di trasporto verificando che il trasporto sia invertibile

- 3) il Tensore della deformazione
- 4) la variazione infinitesima di volume
- 5) il solido deformato
- 6) il Tensore della Tensione (materiale ELOI) con costanti di Lamé  $\lambda$  e  $\mu$ .

Svolgimento

$$a) \underline{f}(x) = \underline{x} + \underline{u}(x) =$$

$$= x_1 \underline{e}_1 + x_2 \underline{e}_2 + x_3 \underline{e}_3 + \underline{u}(x)$$



$$\begin{pmatrix} f_1 \\ f_2 \\ f_3 \end{pmatrix} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} + \begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix} \Rightarrow \begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix} = \begin{pmatrix} f_1 \\ f_2 \\ f_3 \end{pmatrix} - \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

$$\underline{x}' = \underline{f}(\underline{x})$$

$$\underline{f}(x) = \underbrace{(x_1 - 0,2x_2)}_{f_1} \underline{e}_1 + \underbrace{(0,3x_1 + x_2)}_{f_2} \underline{e}_2 + x_3 \underline{e}_3$$

$$\begin{aligned} \underline{u}(x) &= (\cancel{x_1} - 0,2x_2 - \cancel{x_1}) \underline{e}_1 + (0,3x_1 + \cancel{x_2} - \cancel{x_2}) \underline{e}_2 + (\cancel{x_3} - \cancel{x_3}) \underline{e}_3 = \\ &= -0,2x_2 \underline{e}_1 + 0,3x_1 \underline{e}_2 \end{aligned}$$

2) Gradiente di Trasporto

$$\underline{f}(x + dx) = \underline{f}(x) + \underline{F} dx + o(|dx|)$$

$$\underline{F} = \begin{pmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ f_{31} & f_{32} & f_{33} \end{pmatrix}$$

$$\underline{F} = \begin{pmatrix} 1 & -0,2 & 0 \\ 0,3 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\det \underline{F} > 0$$

$$\det \underline{\underline{F}} = 1 + 0 + 0 - (0 - 0,06 + 0) = 1,06 > 0$$

OK

3) Tensore della deformazione

$$\underline{\underline{E}} = \frac{1}{2} (\underline{\underline{H}} + \underline{\underline{H}}^T)$$

$$\underline{\underline{H}} = \begin{pmatrix} u_{11} & u_{12} & u_{13} \\ u_{21} & u_{22} & u_{23} \\ u_{31} & u_{32} & u_{33} \end{pmatrix}$$

$$\underline{\underline{F}} = \underline{\underline{I}} + \underline{\underline{H}} \Rightarrow \underline{\underline{H}} = \underline{\underline{F}} - \underline{\underline{I}}$$

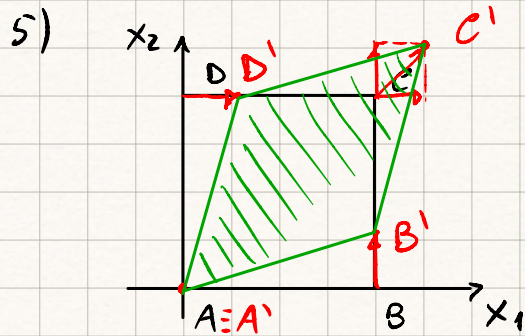
$$\underline{\underline{H}} = \begin{pmatrix} 1 & -0,2 & 0 \\ 0,3 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} - \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & -0,2 & 0 \\ 0,3 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\underline{\underline{E}} = \frac{1}{2} (\underline{\underline{H}} + \underline{\underline{H}}^T) = \frac{1}{2} \left[ \begin{pmatrix} 0 & -0,2 & 0 \\ 0,3 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + \begin{pmatrix} 0 & 0,3 & 0 \\ -0,2 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \right] =$$
$$= \frac{1}{2} \begin{pmatrix} 0 & 0,1 & 0 \\ 0,1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0,05 & 0 \\ 0,05 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

4) Variazione infinitesimale di volume

$$E_V = \text{Tr}(\underline{\underline{E}}) = E_{11} + E_{22} + E_{33}$$

$$E_V = 0 + 0 + 0 = 0$$



$$u(P) = \underline{E} \underline{x}_P$$

$$\begin{aligned} u(\underline{x} + d\underline{x}) &= u(\underline{x}) + \underline{H} d\underline{x} + o(d\underline{x}) \\ &= \underline{u}(\underline{x}) + \underline{E} d\underline{x} + \underline{W} d\underline{x} \end{aligned}$$

$$u(A) = \begin{pmatrix} 0 & 0,05 \\ 0,05 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$u(B) = \begin{pmatrix} 0 & 0,05 \\ 0,05 & 0 \end{pmatrix} \begin{pmatrix} L \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0,05L \end{pmatrix}$$

$$u(C) = \begin{pmatrix} 0 & 0,05 \\ 0,05 & 0 \end{pmatrix} \begin{pmatrix} L \\ L \end{pmatrix} = \begin{pmatrix} 0,05L \\ 0,05L \end{pmatrix}$$

$$u(D) = \begin{pmatrix} 0 & 0,05 \\ 0,05 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ L \end{pmatrix} = \begin{pmatrix} 0,05L \\ 0 \end{pmatrix}$$

6) Tensore della deformazione

$$\underline{\underline{T}} = 2\mu \underline{\underline{E}} + \lambda (\text{Tr} \underline{\underline{E}}) \underline{\underline{I}}$$

$$\underline{\underline{T}} = 2 \cdot 23027 \cdot \begin{pmatrix} 0 & 0,05 & 0 \\ 0,05 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + 34615 \cdot (0 + 0 + 0) \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} =$$

$$= \begin{pmatrix} 0 & 2307,7 & 0 \\ 2307,7 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \Pi P_2$$