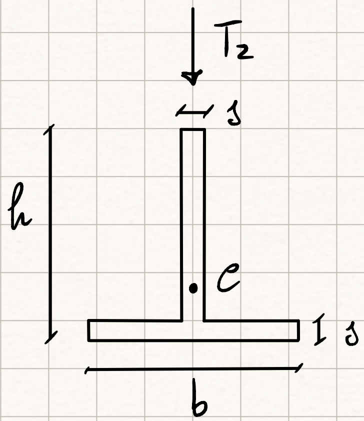
Esercizio



$T_z = 20$ KN F = 10 KN (di trazione)

$h = b = 20$ cm $s = 2$ cm

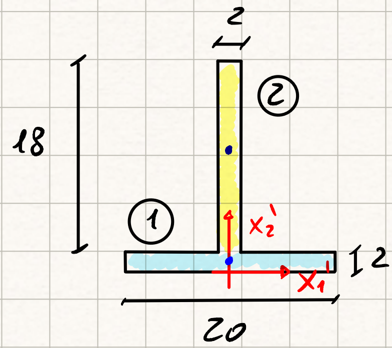
1) Determinare le tensioni sulla sezione
2) Calcolare le tensioni principali nel punto maggiormente sollecitato
3) Determinare il tensore della deformazione nel punto maggiormente sollecitato

($E = 30000$ MPa ; $\nu = 0,2$)

Svolgimento

Calcoliamo le proprietà geometriche della sezione.

$$A = (18\cdot 2 + 20\cdot 2)\ cm^2 = 76\ cm^2$$

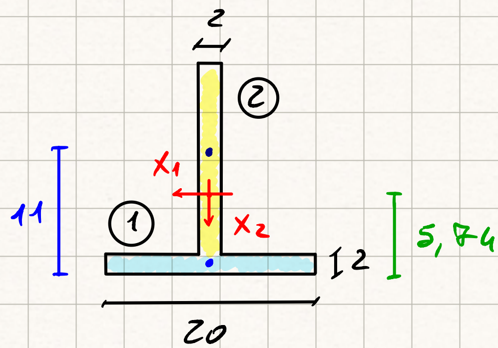


Baricentro:

$$x_{1G} = 0$$

$$x_{2G} = \frac{20\cdot 2\cdot 1 + 18\cdot 2\cdot 11}{76} = 5{,}74\ cm$$

Momento di inerzia



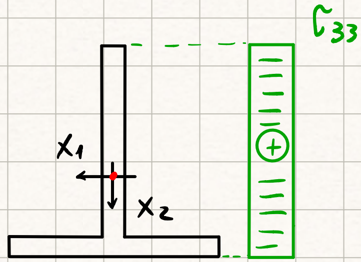
$$I_1^{(2)} = \frac{1}{12}\cdot 2\cdot 18^3 + 36(-5{,}26)^2 = 1968{,}03\ cm^4$$

$$I_1^{(1)} = \frac{1}{12}\cdot 20\cdot 2^3 + 40\,(4{,}74)^2 = 912{,}04\ cm^4$$

$$I_1 = I_1^{(1)} + I_1^{(2)} = 2880{,}07 \ cm^4$$

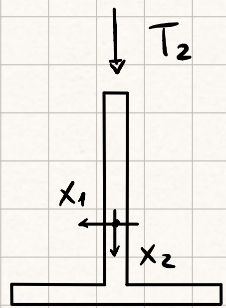
Determiniamo le Tensioni assiali.

$$N = F = 10 \ KN$$

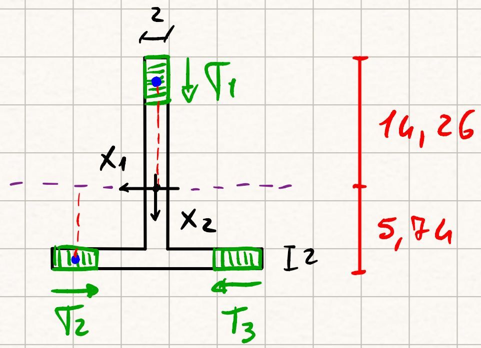


$$\sigma_{33} = \frac{N}{A} = \frac{10 \ KN}{76 \ cm^2} = 0{,}13 \ \frac{KN}{cm^2} = 1{,}3 \ MPa$$

Determiniamo le Tensioni Tangenziali.



$$\tau_{3i} = - \frac{T_2 \, S_1^*}{I_1 \, \delta_j}$$



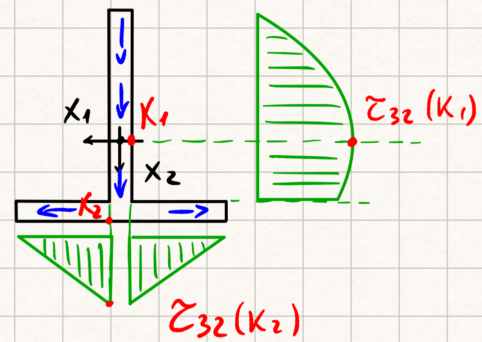
$$S_1^{*(1)} = -2\xi_1\left(14{,}26 - \frac{\xi_1}{2}\right)$$

$$S_1^{*(2)} = 2\xi_2\,(5{,}74 - 1)$$

$$S_1^{*(3)} = S_1^{*(2)}$$

$$\tau_{32}^{(1)} = -\frac{20\cdot\left[-2\xi_1\left(14{,}26 - \frac{\xi_1}{2}\right)\right]}{2880{,}07\cdot 2}$$

$$\tau_{31}^{(2)} = \tau_{31}^{(3)} = -\frac{20\cdot\left[2\xi_2\cdot 4{,}74\right]}{2880{,}07\cdot 2}$$



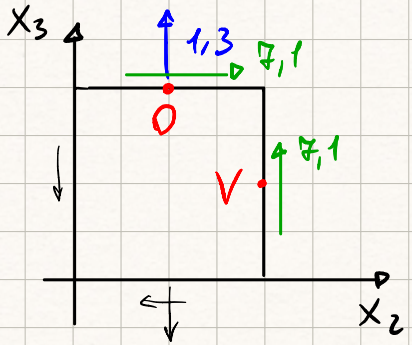
$$\tau_{31}(K_1) = \frac{20\left[14{,}26\cdot\left(14{,}26 - \frac{14{,}26}{2}\right)\right]}{2880{,}07} = 0{,}71\ \frac{KN}{cm^2} = 7{,}1\ MPa$$

$$\tau_{32}(K_2) = -\frac{20\,[3 \cdot 4{,}74]}{2880{,}07} = -0{,}3\ \frac{KN}{cm^2} = -3\ MPa$$

Il punto più sollecitato della sezione è K_1.

2) In K_1: $\sigma_{33} = 1{,}3\ MPa \qquad \tau_{32} = 7{,}1\ MPa \qquad \tau_{31} = 0$

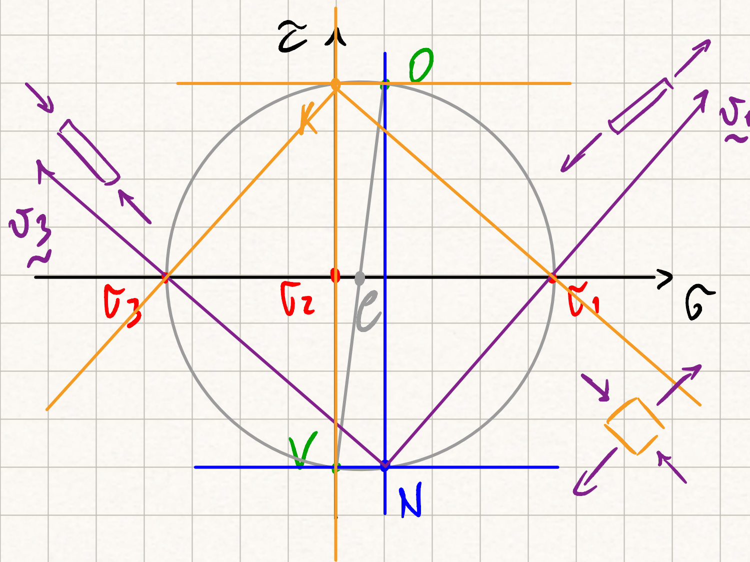
$$\underline{\underline{T}}(K_1) = \begin{pmatrix} \sigma_{11} & \tau_{21} & \tau_{31} \\ \tau_{12} & \sigma_{22} & \tau_{32} \\ \tau_{13} & \tau_{23} & \sigma_{33} \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 7{,}1 \\ 0 & 7{,}1 & 1{,}3 \end{pmatrix} MPa$$



x_3
1,3
7,1
O
V
7,1
x_2

$O(1{,}3;\ 7{,}1)$

$V(0,\ -7{,}1)$



τ
O
$\underset{\sim}{\sigma_1}$
$\underset{\sim}{\sigma_3}$
σ_3
σ_2
C
σ_1
σ
V
N

$$\sigma_{1,3} = \frac{\sigma_o + \sigma_v}{2} \pm \sqrt{\left(\frac{\sigma_o - \sigma_v}{2}\right)^2 + \tau^2} = \frac{1,3}{2} \pm \sqrt{\left(\frac{1,3}{2}\right)^2 + 7,1^2} =$$

$$= 0,65 \pm 7,13 = \begin{cases} \sigma_1 = 7,78 \text{ MPa} \\ \sigma_3 = -6,48 \text{ MPa} \end{cases}$$

$$\theta = \frac{1}{2} \operatorname{arctg}\left(\frac{2\tau v}{\sigma_v - \sigma_o}\right) = \frac{1}{2} \operatorname{arctg}\left(\frac{-2 \cdot 7,1}{-1,3}\right) = 0,77 \rightarrow 44^\circ$$

3) $\underline{\underline{\varepsilon}} = \dfrac{1+\nu}{E}\,\underline{\underline{T}} - \dfrac{\nu}{E}(\mathrm{Tr}\,\underline{\underline{T}})\,\underline{\underline{I}} =$

$$= \frac{1+0{,}2}{30000}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 7{,}1 \\ 0 & 7{,}1 & 1{,}3 \end{pmatrix} - \frac{0{,}2}{30000}(1{,}3)\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} =$$

$$= 4\cdot 10^{-5}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 7{,}1 \\ 0 & 7{,}1 & 1{,}3 \end{pmatrix} - 0{,}86\cdot 10^{-5}\cdot\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} =$$

$$= \begin{pmatrix} -0{,}86 & 0 & 0 \\ 0 & -0{,}86 & 28{,}4 \\ 0 & 28{,}4 & 4{,}34 \end{pmatrix}\cdot 10^{-5}$$