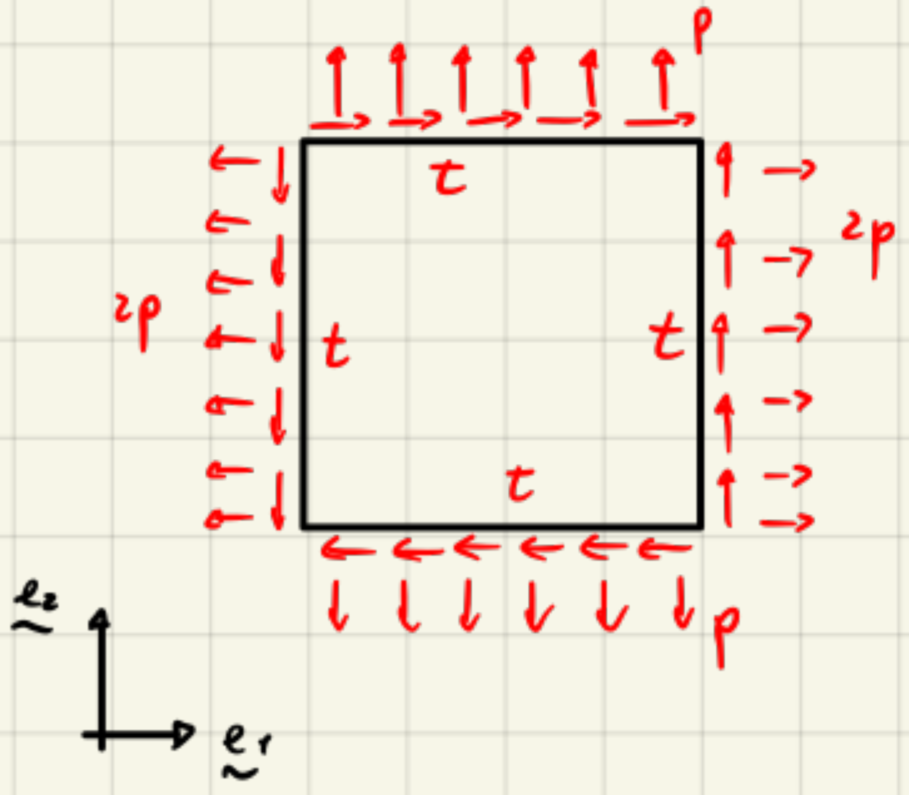
Esempio 1

Dato il pannello in figura, determinare



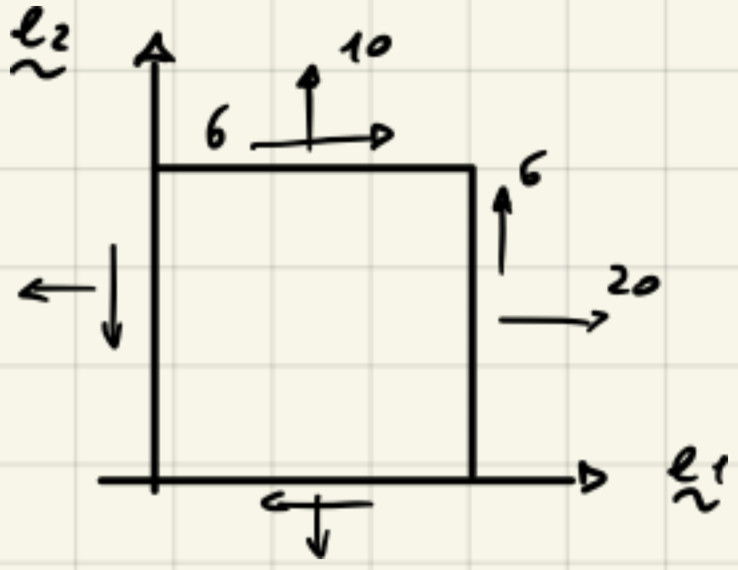
1) il Tensore degli sforzi;
2) le Tensioni e le direzioni principali di Tensione;
3) il Tensore della deformazione considerando il materiale ELOI.

$p = 10$ MPa , $t = 6$ MPa

$E = 210 \cdot 10^3$ MPa , $\nu = 0,2$

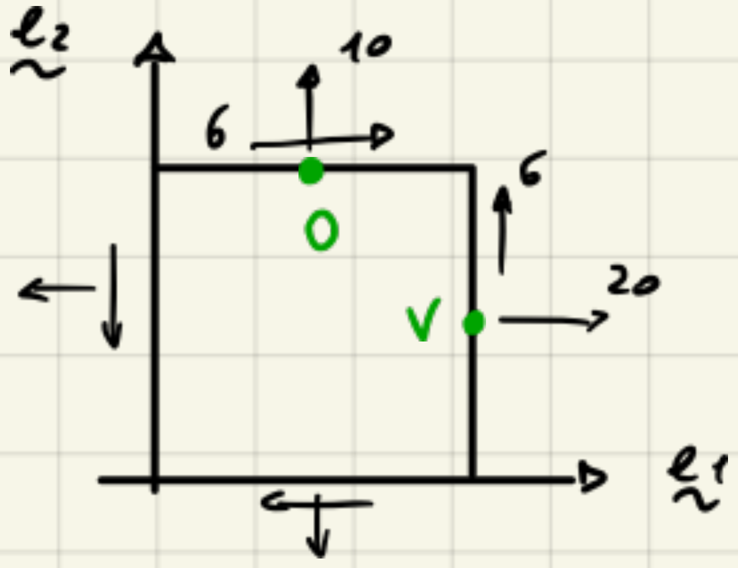
• Svolgimento

1)

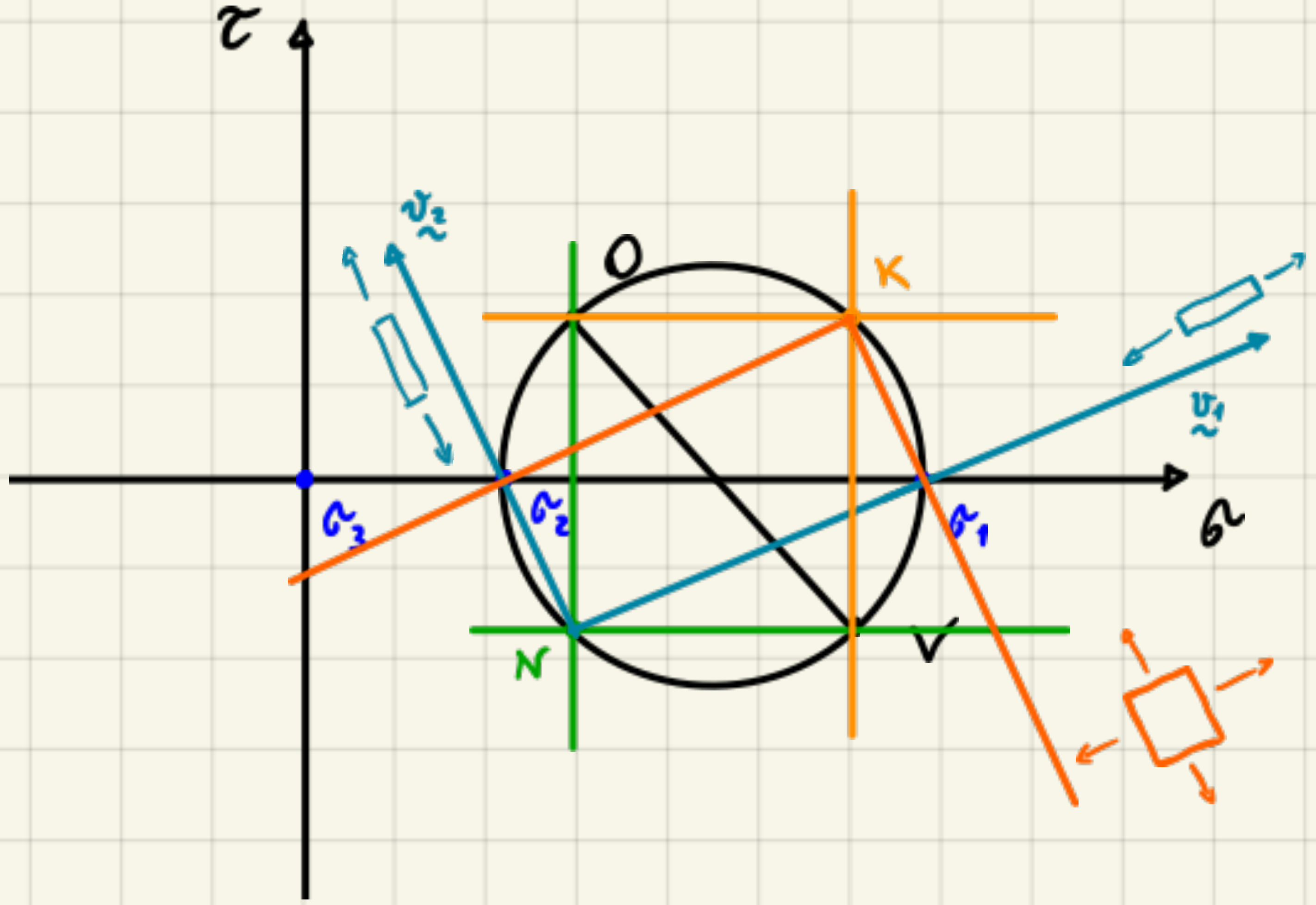


$$\underline{\underline{T}} = \begin{pmatrix} 20 & 6 & 0 \\ 6 & 10 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

2)



O (10, 6)
V (20, -6)



$$\sigma_{1,2} = \frac{\sigma_O + \sigma_V}{2} \pm \sqrt{\left(\frac{\sigma_O - \sigma_V}{2}\right)^2 + \tau^2} = \frac{10+20}{2} \pm \sqrt{\left(-\frac{10}{2}\right)^2 + 6^2} =$$

$$= 15 \pm \sqrt{25+36} = 15 \pm \sqrt{61} = \begin{cases} 22,81 \text{ MPa} \\ 7,19 \text{ MPa} \end{cases}$$

$$\theta = \frac{1}{2} \operatorname{arctg}\left(\frac{2\tau}{\sigma_O - \sigma_V}\right) = \frac{1}{2} \operatorname{arctg}\left(\frac{12}{10}\right) = 25,09$$

3)

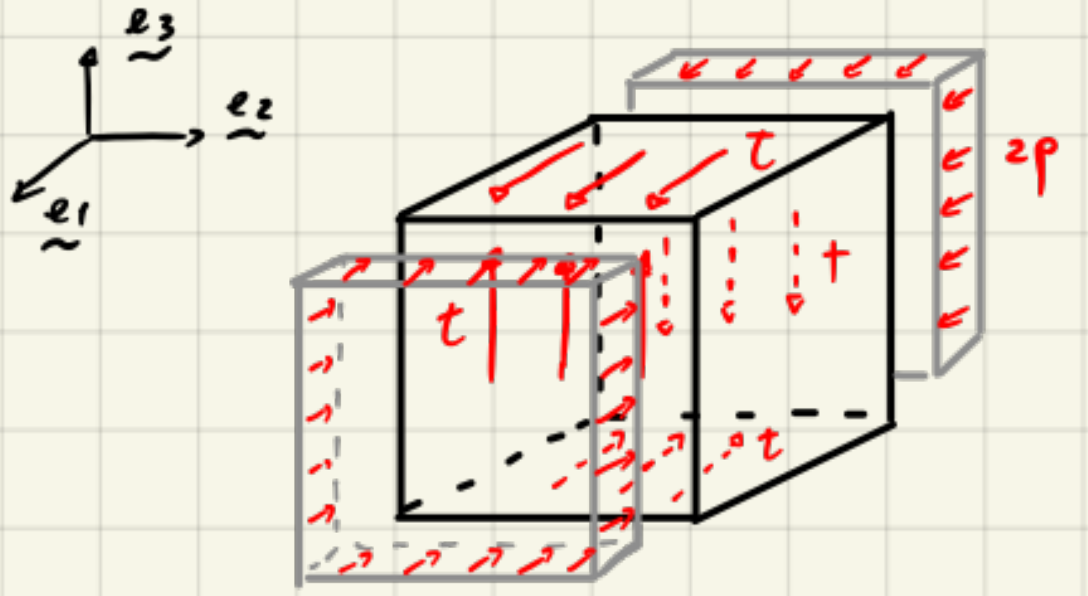
$$\underline{\underline{E}} = \frac{1+\nu}{E} \underline{\underline{T}} - \frac{\nu}{E} \left(\mathrm{tr}\, \underline{\underline{T}} \right) \underline{\underline{I}} =$$

$$= \frac{1,2}{210 \cdot 10^3} \begin{pmatrix} 20 & 6 & 0 \\ 6 & 10 & 0 \\ 0 & 0 & 0 \end{pmatrix} - \frac{0,2}{210 \cdot 10^3} \left(20 + 10 + 0 \right) \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} =$$

$$= \frac{1}{210 \cdot 10^3} \left[\begin{pmatrix} 24 & 7,2 & 0 \\ 7,2 & 12 & 0 \\ 0 & 0 & 0 \end{pmatrix} - \begin{pmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{pmatrix} \right] =$$

$$= \frac{1}{210 \cdot 10^3} \begin{pmatrix} 18 & 7,2 & 0 \\ 7,2 & 6 & 0 \\ 0 & 0 & -6 \end{pmatrix}$$

Esempio 2



$\underset{\sim}{f_1} = -2p\,\underset{\sim}{e_1} + \tau\,\underset{\sim}{e_3}$ su ∂C_{1+}

$\underset{\sim}{f_2} = \underset{\sim}{0}$ su ∂C_{2+}

$\underset{\sim}{f_3} = \tau\,\underset{\sim}{e_1}$ su ∂C_{3+}

Forze di volume nulle.

Dato il cubo di C di lato ℓ, determinare:

1) lo stato di tensione; riconoscere che è piano e individuare tale piano;

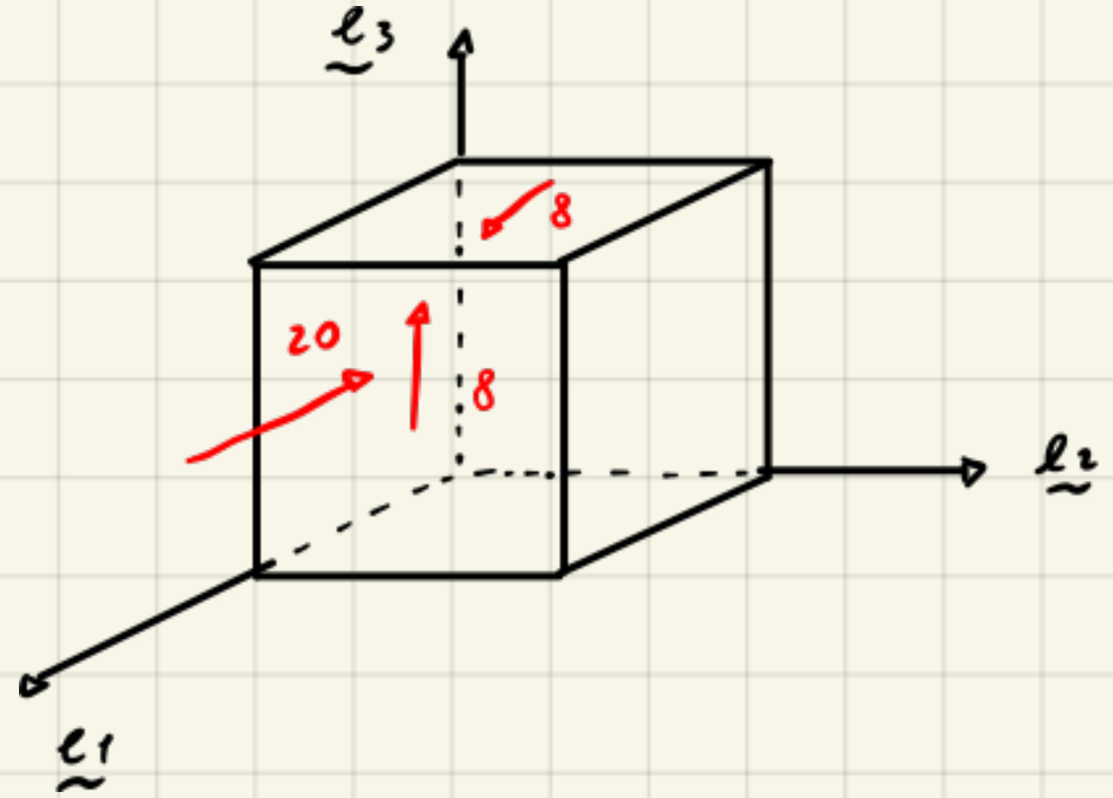
2) tensioni principali e direzioni principali di tensione;

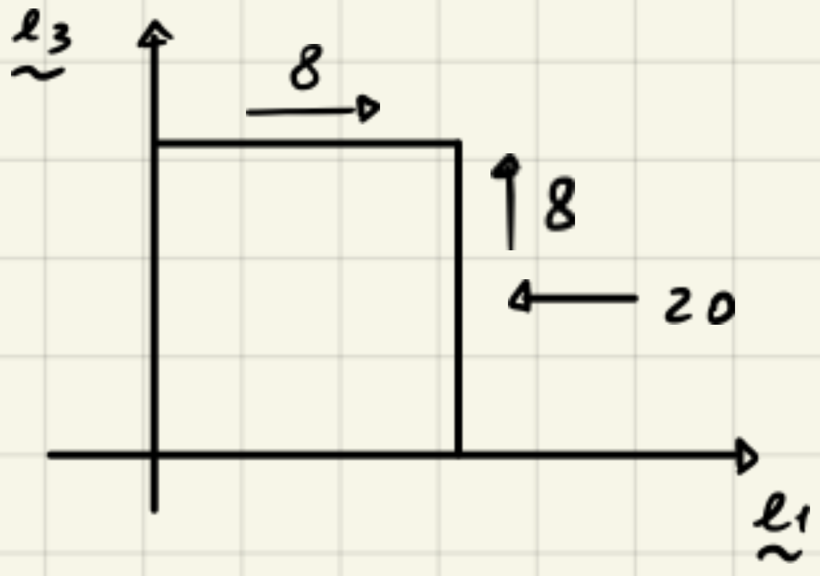
3) lo stato di deformazione considerando il materiale ELOI.

$\ell = 100$ mm; $p = 10$ MPa; $\tau = 8$ MPa; $E = 40 \cdot 10^3$ MPa; $\nu = 0{,}2$

· Svolgimento

1) Ridisegno il cubo in modo più "pulito":





Dall'equilibrio esterno sappiamo che $\underset{\sim}{t_n} = \underset{\sim}{f_n}$;

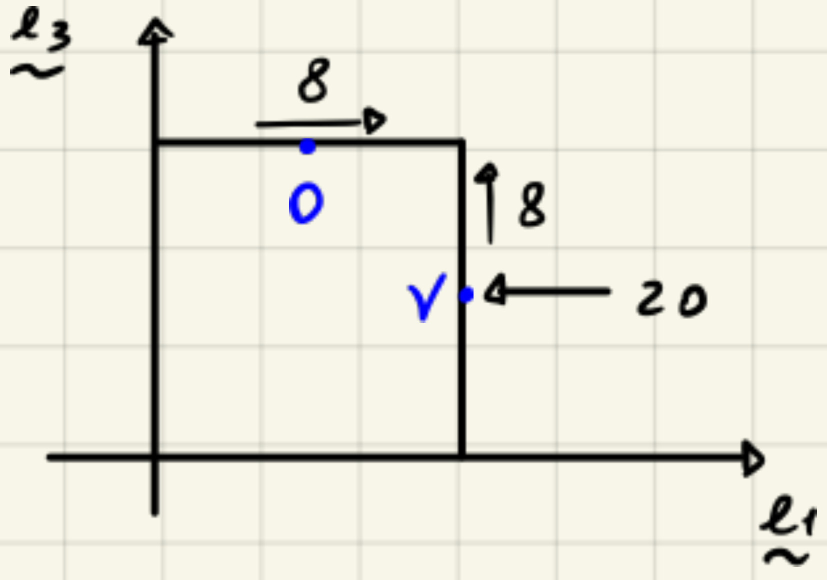
dal Teorema di Cauchy si ha che: $\underset{\sim}{t_n} = \underline{\underline{T}}\,\underset{\sim}{n}$.

Dunque possiamo scrivere:

$$\underline{\underline{T}} = \left(\underset{\sim}{t_1} \;\; \underset{\sim}{t_2} \;\; \underset{\sim}{t_3} \right) = \left(\underset{\sim}{f_1} \;\; \underset{\sim}{f_2} \;\; \underset{\sim}{f_3} \right) = \begin{pmatrix} -2p & 0 & \tau \\ 0 & 0 & 0 \\ \tau & 0 & 0 \end{pmatrix} = \begin{pmatrix} -20 & 0 & 8 \\ 0 & 0 & 0 \\ 8 & 0 & 0 \end{pmatrix}$$

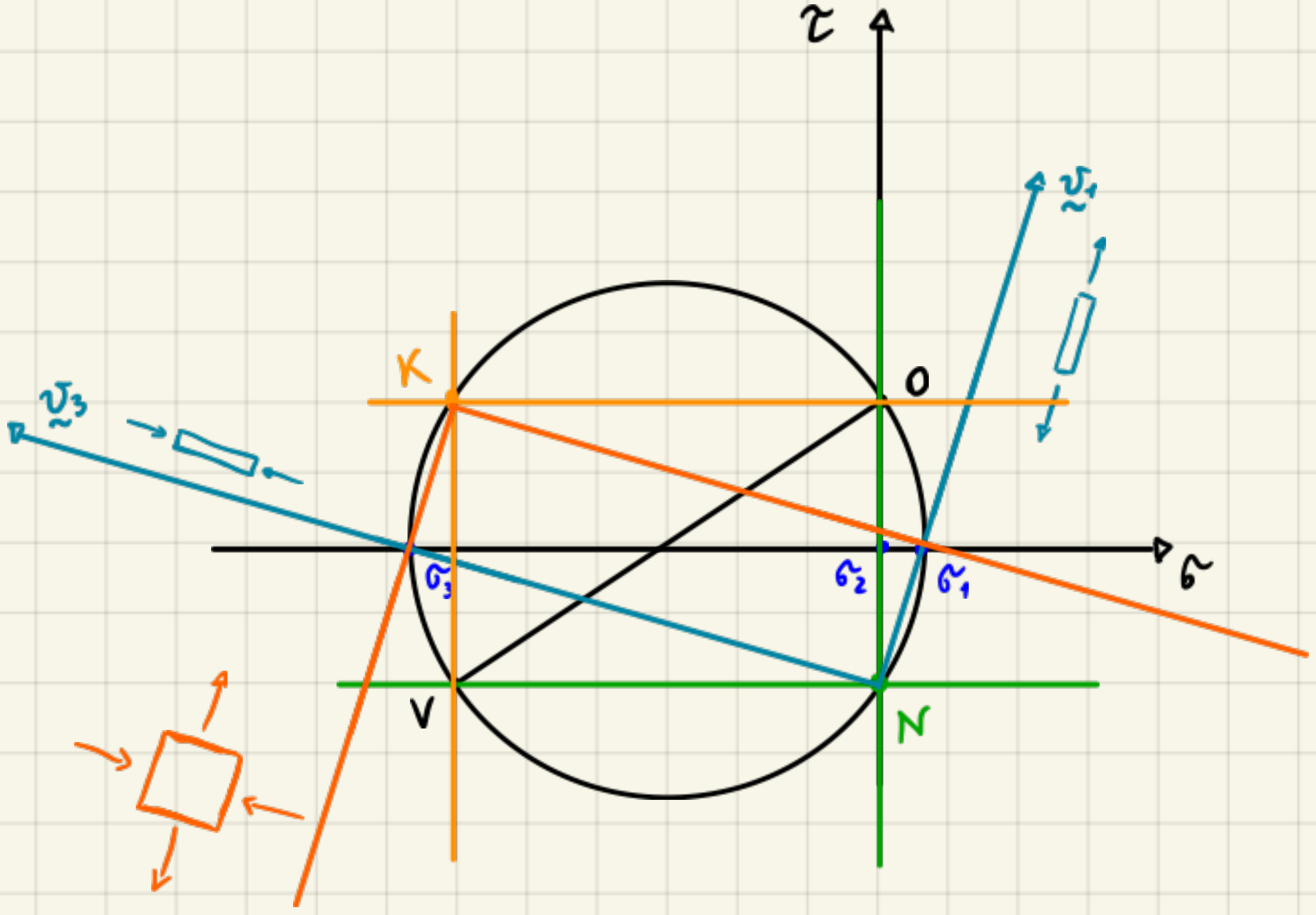
$\det(\underline{\underline{T}}) = 0 \Rightarrow$ stato di tensione piano $\Rightarrow \underset{\sim}{e_1} - \underset{\sim}{e_3}$

2)



O (0, 8)

V (-20, -8)



$$\sigma_{1,3} = \frac{\sigma_O + \sigma_V}{2} \pm \sqrt{\left(\frac{\sigma_O - \sigma_V}{2}\right)^2 + \tau^2} = \frac{-20}{2} \pm \sqrt{100 + 64} =$$

$$= -10 \pm 12,8 \begin{cases} \sigma_1 = 2,8 \text{ MPa} \\ \sigma_3 = -22,8 \text{ MPa} \end{cases}$$

$$\theta = \frac{1}{2} \operatorname{arctg}\left(\frac{2\tau}{\sigma_O - \sigma_V}\right) = \frac{1}{2} \operatorname{arctg}\left(\frac{-16}{20}\right) = -19,32$$

3)

$$\underset{=}{E} = \frac{1+\nu}{E}\, \underset{=}{T} - \frac{\nu}{E} \left(\operatorname{tr} \underset{=}{T}\right) \underset{=}{I} =$$

$$= \frac{1,2}{40 \cdot 10^3} \begin{pmatrix} -20 & 0 & 8 \\ 0 & 0 & 0 \\ 8 & 0 & 0 \end{pmatrix} - \frac{0,2}{40 \cdot 10^3} \cdot (-20) \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} =$$

$$= \frac{1}{40 \cdot 10^3} \left[\begin{pmatrix} -24 & 0 & 9,6 \\ 0 & 0 & 0 \\ 9,6 & 0 & 0 \end{pmatrix} + \begin{pmatrix} 24 & 0 & 0 \\ 0 & 24 & 0 \\ 0 & 0 & 24 \end{pmatrix} \right] =$$

$$= \frac{1}{40 \cdot 10^3} \begin{pmatrix} 0 & 0 & 9,6 \\ 0 & 24 & 0 \\ 9,6 & 0 & 24 \end{pmatrix}$$