Esercizio

Data la funzione $f(x, y) = \ln[(x - 2)(y + 3)]$ calcolare:

- il dominio di f;
- il piano tangente in $P_1 = (3, e - 3)$;
- la derivata direzionale in $P_2 = (8,0)$ lungo la direzione della bisettrice del primo e terzo quadrante del piano nel verso delle x decrescenti.

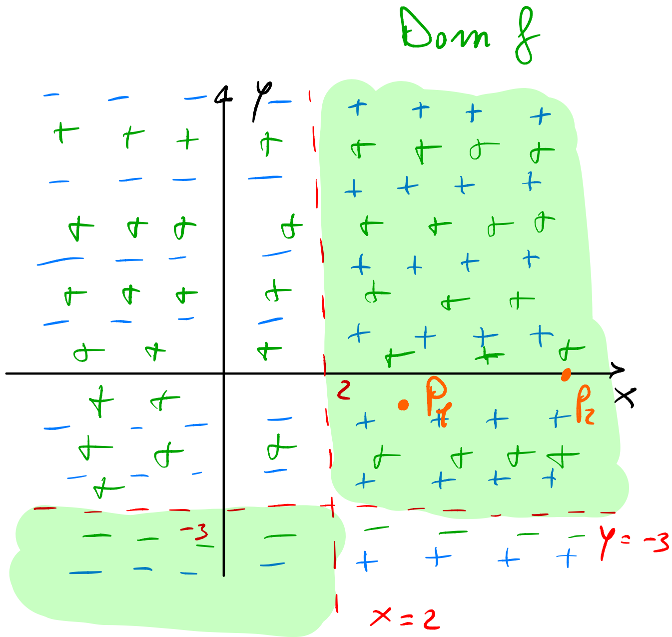
1) Dominio

$$f(x,y) = \ln\left[(x-2)(y+3)\right]$$

$$(x-2)(y+3) > 0 \quad \begin{cases} (+)\cdot(+) \\ (-)\cdot(-) \end{cases}$$

$x - 2 > 0 \Rightarrow \underline{x > 2}$

$y + 3 > 0 \Rightarrow \underline{y > -3}$



2) Piano Tangente in $P_1(3, e-3)$

$P_1 \in \text{Dom} f \Rightarrow f$ è continua in P_1.

$$f_x(x,y) = \frac{1}{(x-2)\cancel{(y+3)}} \cdot \cancel{(y+3)} = \frac{1}{x-2}$$

$$f_y(x,y) = \frac{1}{\cancel{(x-2)}(y+3)} \cdot \cancel{(x-2)} = \frac{1}{y+3}$$

f_x e f_y sono continue in Dom f $\Rightarrow$ $f \in C^1(P_1)$ $\Rightarrow$
$\Rightarrow$ f è diff.bile in P_1 $\Rightarrow$ esiste piano tangente in P_1.

$$z = f(x_0,y_0) + f_x(x_0,y_0)(x-x_0) + f_y(x_0,y_0)(y-y_0)$$

$$f(x,y) = \ln[(x-2)(y+3)] \qquad f_x(x,y) = \frac{1}{x-2} \qquad f_y(x,y) = \frac{1}{y+3}$$

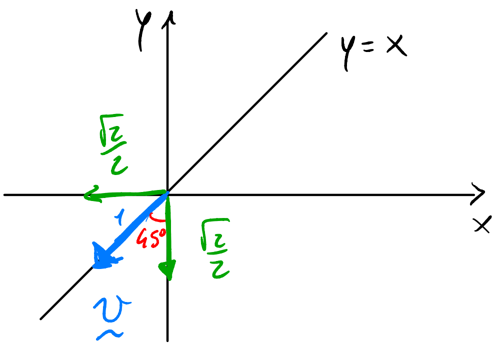
$$f(3, e-3) = \ln[(3-2)(e-\cancel{3}+\cancel{3})] = \ln(e) = 1$$

$$f_x(3, e-3) = \frac{1}{3-2} = 1 \qquad f_y(3, e-3) = \frac{1}{e-3+3} = \frac{1}{e}$$

$$z = 1 + 1\cdot(x-3) + \frac{1}{e}(y-e+3) \Rightarrow z = \cancel{1} + x - 3 + \frac{1}{e}y \cancel{-1} + \frac{3}{e} \Rightarrow$$

$$\Rightarrow \boxed{z = x + \frac{1}{e}y - 3 + \frac{3}{e}}$$

3) Derivata direzionale in $P_2(8,0)$



$$\underset{\sim}{v} = \begin{pmatrix} -\frac{\sqrt{2}}{2} \\ -\frac{\sqrt{2}}{2} \end{pmatrix}$$

$f \in C^1(P_2) \Rightarrow f$ è diff.bile in $P_2 \Rightarrow$

$$\frac{\partial f}{\partial \underset{\sim}{v}}(P_2) = \langle \underset{\sim}{\nabla} f(P_2), \underset{\sim}{v} \rangle$$

$$f_x(x,y) = \frac{1}{x-2} \Rightarrow f_x(8,0) = \frac{1}{6}$$

$$f_y(x,y) = \frac{1}{y+3} \Rightarrow f_y(8,0) = \frac{1}{3}$$

$$\Rightarrow \underset{\sim}{\nabla} f(8,0) = \begin{pmatrix} 1/6 \\ 1/3 \end{pmatrix}$$

$$\frac{\partial f}{\partial \underset{\sim}{v}}(8,0) = \left\langle \begin{pmatrix} 1/6 \\ 1/3 \end{pmatrix}, \begin{pmatrix} -\frac{\sqrt{2}}{2} \\ -\frac{\sqrt{2}}{2} \end{pmatrix} \right\rangle = -\frac{\sqrt{2}}{12} - \frac{\sqrt{2}}{6} = -\frac{\overset{1}{\cancel{3}}}{\underset{4}{\cancel{12}}}\sqrt{2} = -\frac{\sqrt{2}}{4}$$