Proposizione

Sia $\mathbf{F}$ un campo vettoriale definito e continuo in $\Omega \subseteq \mathbb{R}^n$.

Sono condizioni equivalenti:

1) date comunque due curve (regolari o regolari a tratti) γ_1 e γ_2 con sostegno in Ω e estremi coincidenti, risulta:

$$\int_{\gamma_1} \mathbf{F} \cdot d\gamma = \pm \int_{\gamma_2} \mathbf{F} \cdot d\gamma$$

2) per ogni curva chiusa regolare (a tratti) e con sostegno in Ω si ha:

$$\oint_{\gamma} \mathbf{F} \cdot d\gamma = 0$$

Dimostrazione

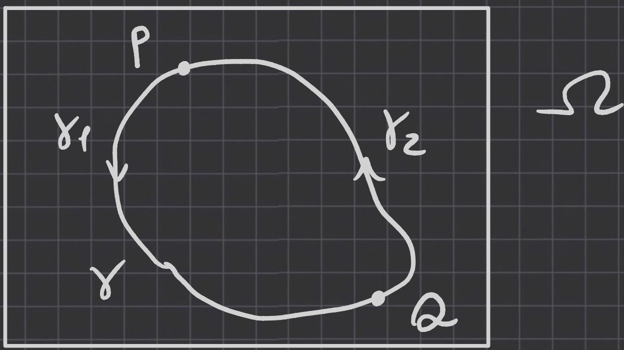
(1) <=> (2) (1) => (2) (2) => (1)

=>)

$\underset{\sim}{\gamma} : [a, b] \to \Omega$ t.c. $\gamma(a) = \gamma(b)$

P, Q $(P \neq Q)$ t.c. $\gamma(t_1) = P$ e $\gamma(t_2) = Q$

con $t_1 \neq t_2$.



$$\gamma = \gamma_1 \cup \gamma_2$$

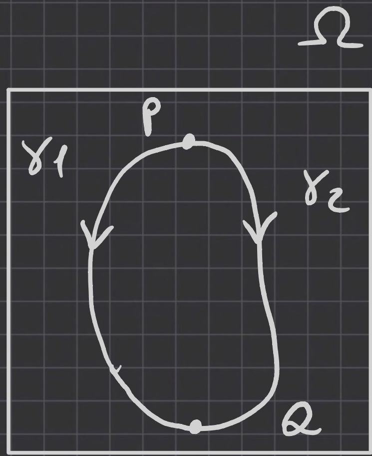
$$\int_{\gamma_1} \underset{\sim}{F} \cdot d\underset{\sim}{\gamma} = - \int_{\gamma_2} \underset{\sim}{F} \cdot d\gamma \Rightarrow$$

$$\Rightarrow \int_{\gamma} \underset{\sim}{F} \cdot d\underset{\sim}{\gamma} = \int_{\gamma_1} \underset{\sim}{F} \cdot d\gamma + \int_{\gamma_2} \underset{\sim}{F} \cdot d\gamma = 0 \Rightarrow$$

$$\Rightarrow \oint_{\gamma} \underset{\sim}{F} \cdot d\gamma = 0$$

$\Leftarrow)$

$\gamma = \gamma_1 \cup (-\gamma_2)$



$$\oint_{\gamma} \underset{\sim}{F} \cdot d\gamma = 0 \Rightarrow$$

$$\Rightarrow \int_{\gamma_1 \cup (-\gamma_2)} \underset{\sim}{F} \cdot d\underset{\sim}{\gamma} = \int_{\gamma_1} \underset{\sim}{F} \cdot d\underline{\gamma} + \int_{-\gamma_2} \underset{\sim}{F} \cdot d\underline{\gamma} =$$

$$= \int_{\gamma_1} \underset{\sim}{F} \cdot d\underset{\sim}{\gamma} - \int_{\gamma_2} \underset{\sim}{F} \cdot d\underset{\sim}{\gamma} = 0 \Rightarrow$$

$$\Rightarrow \int_{\gamma_1} \underset{\sim}{F} \cdot d\underset{\sim}{r} = \int_{\gamma_2} \underset{\sim}{F} \cdot d\underset{\sim}{r}$$

$(1) \Rightarrow (2)$ $(2) \Rightarrow (1)$

$$(1) \Leftrightarrow (2)$$