Prop

Sia $\underset{\sim}{F} = \nabla \underset{\sim}{U}$ un campo conservativo definito e continuo in $\Omega \subseteq \mathbb{R}^n$.

Per ogni curva $\gamma : [a,b] \to \Omega$ regolare, si ha:

$$\int_\gamma \underset{\sim}{F} \cdot d\underset{\sim}{\gamma} = U(\gamma(b)) - U(\gamma(a)).$$

Dimostrazione

$$\int_\gamma \underset{\sim}{F} \cdot d\underset{\sim}{\gamma} = \int_a^b \underset{\sim}{F}(\underset{\sim}{\gamma}(t)) \cdot \underset{\sim}{\gamma}'(t)\, dt$$

$$\underset{\sim}{F} = \underset{\sim}{\nabla} U$$

$$\underset{\sim}{F}(\underline{\gamma}(t)) \cdot \underset{\sim}{\gamma}'(t) = \frac{d}{dt} U(\gamma(t))$$

$$\int_a^b \underset{\sim}{F}(\underset{\sim}{r}(t)) \cdot \underset{\sim}{r}'(t)\, dt =$$

$$= \int_a^b \frac{d}{dt} U(r(t))\, dt = U(r(t)) \Big|_a^b =$$

$$= U(r(b)) - U(r(a))$$