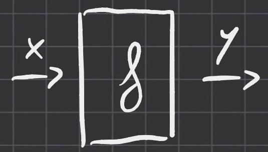
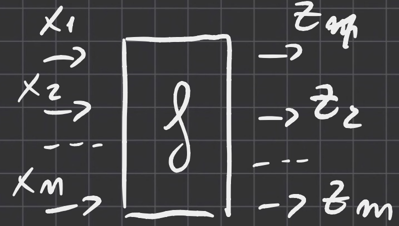
Campi vettoriali in $\mathbb{R}^2$ e $\mathbb{R}^3$

$f : A\,(\subseteq \mathbb{R}) \to B\,(\subseteq \mathbb{R})$



$y = f(x)$

$f : A\,(\subseteq \mathbb{R}^n) \to B\,(\subseteq \mathbb{R}^m)$ $\quad n = m \quad n > 1$

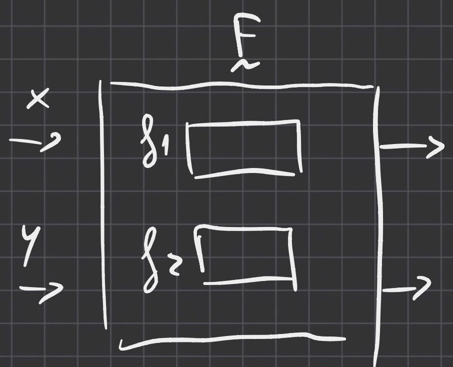


$n = m$

$n = m = 2$

$n = m = 3$

$\underset{\sim}{F} : A\,(\subseteq \mathbb{R}^2) \to B\,(\subseteq \mathbb{R}^2)$ $\qquad \underset{\sim}{F}(x,y) = (f_1(x,y), f_2(x,y))$



$z_1 = f_1(x,y)$

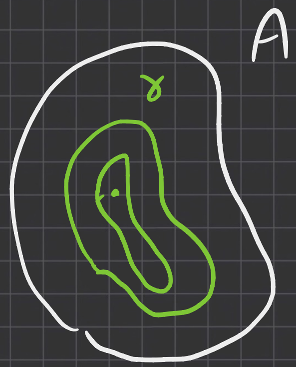
$z_2 = f_2(x,y)$

Dominio, Rotazionalità, Conservatività

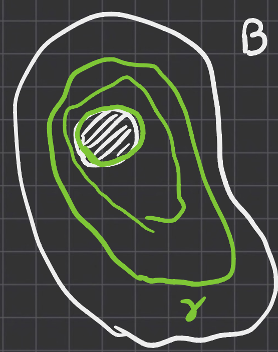
1) Dominio

$$\mathrm{Dom}\,\underset{\sim}{F} = \begin{cases} \mathrm{Dom}\, f_1 \\ \mathrm{Dom}\, f_2 \end{cases}$$

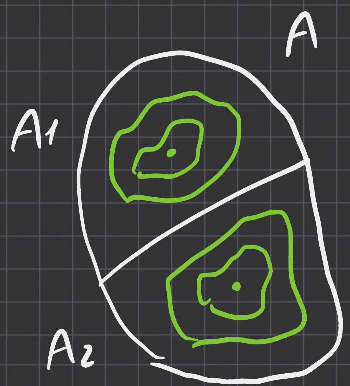
Oss. 1



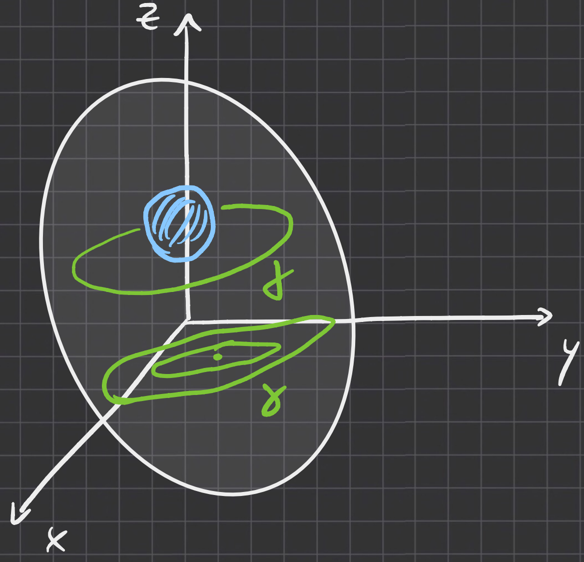
Semplicemente Connesso

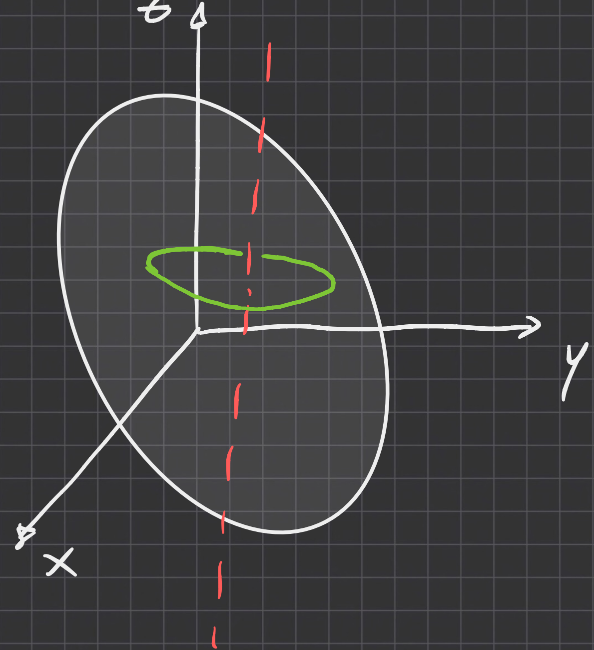


Non è semplicemente Connesso



$$A = A_1 \cup A_2$$





2) Rotazionalità

$$\operatorname{rot} \underset{\sim}{F} = \frac{\partial f_2}{\partial x} - \frac{\partial f_1}{\partial y}$$

$$\operatorname{rot} \underset{\sim}{F} = 0 \Rightarrow \frac{\partial f_2}{\partial x} = \frac{\partial f_1}{\partial y} \Rightarrow \underset{\sim}{F} \text{ è irrotazionale}$$

Oss 2

$\underset{\sim}{F} : A (\subseteq \mathbb{R}^3) \to \mathbb{R}^3$

$$\text{rot}\, \underset{\sim}{F} = \underset{\sim}{\nabla} \times \underset{\sim}{F} = \det \begin{pmatrix} \underset{\sim}{i} & \underset{\sim}{j} & \underset{\sim}{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ f_1 & f_2 & f_3 \end{pmatrix} = \left(\frac{\partial f_3}{\partial y} - \frac{\partial f_2}{\partial z} \right) \underset{\sim}{i} + \left(\frac{\partial f_1}{\partial z} - \frac{\partial f_3}{\partial x} \right) \underset{\sim}{j} + \left(\frac{\partial f_2}{\partial x} - \frac{\partial f_1}{\partial y} \right) \underset{\sim}{k}$$

$$\text{rot}\, \underset{\sim}{F} = 0 \Rightarrow \begin{aligned} f_{3y} &= f_{2z} \\ f_{1z} &= f_{3x} \\ f_{2x} &= f_{1y} \end{aligned}$$

3) Conservatività

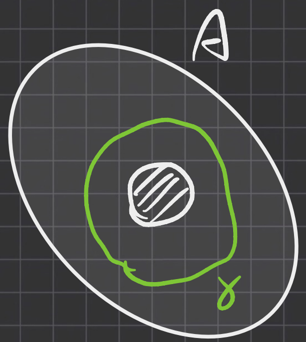
$U : A (\subseteq \mathbb{R}^2) \to \mathbb{R}$ t.c. $\underset{\sim}{\nabla} U = \underset{\sim}{F} \Rightarrow \frac{\partial U}{\partial x} = f_1(x,y)$; $\frac{\partial U}{\partial y} = f_2(x,y)$

Irrotaz. è condizione necessaria per la conservatività

Oss 3

Se $\underset{\sim}{F}$ è irrot. su un insieme semplicemente connesso $\Rightarrow$ esiste il potenziale U di $\underset{\sim}{F}$ $\Rightarrow$ $\underset{\sim}{F}$ è conservativo

Oss 4

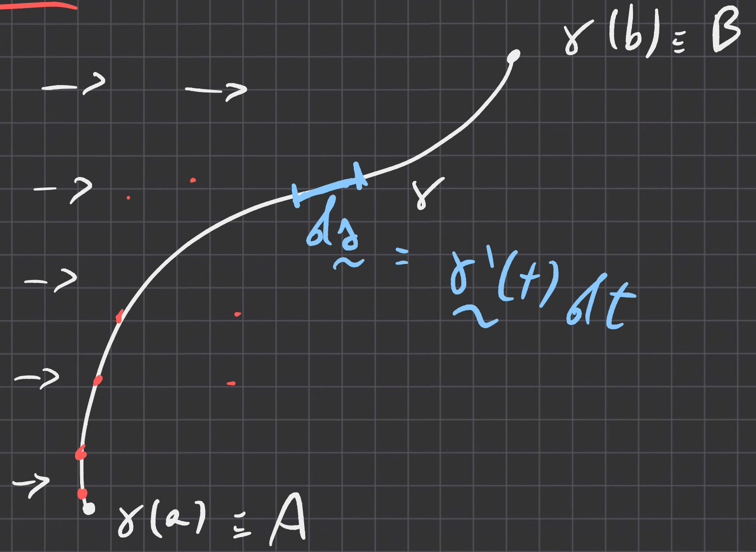


$$\oint_{\gamma} \underset{\sim}{F} \cdot d\underset{\sim}{s} = 0$$

Lavoro di un campo vettoriale lungo una curva (integrale curvilineo di II specie)

$\underset{\sim}{\gamma} : [a,b] \to \mathbb{R}^2,$ $$\int_\gamma \underset{\sim}{F} \cdot d\underset{\sim}{s} = \int_a^b \underset{\sim}{F}(\underset{\sim}{\gamma}(t)) \cdot \underset{\sim}{\gamma}'(t)\, dt$$

Oss 5



$$\mathcal{L} = \underset{\sim}{F} \cdot \underset{\sim}{s}$$

$$d\mathcal{L} = \underset{\sim}{F} \cdot d\underset{\sim}{s} = \underset{\sim}{F} \cdot \underset{\sim}{\gamma}'(t)\, dt$$

$$\mathcal{L} = \int_a^b \underset{\sim}{F}(\underset{\sim}{\gamma}(t)) \cdot \underset{\sim}{\gamma}'(t)\, dt$$

$\underset{\sim}{F}(x,y)$

Oss 6

Se $\underset{\sim}{F}$ é conservativo il lavoro svolto da $\underset{\sim}{F}$ per passare dal punto A al punto B é dato dalla diff.za di potenziale:

$$\mathcal{L}_{AB} = \int_a^b \underset{\sim}{F}(\underset{\sim}{\gamma}(t)) \cdot \underset{\sim}{\gamma}'(t)\, dt = U(B) - U(A) = U(\gamma(b)) - U(\gamma(a))$$